Annualization

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Goals

- **Use ACES simulation outputs to assess alternatives through**
  - Annualization: converting results for a few days to yearly estimates
  - Valuation: assessing relative value of different alternatives based on a large set of performance metrics
- **Present focus: annualization**

Overview

- **Challenge of developing annual estimates of concept metrics from a small number of ACES runs**
- **Two tracks**
  - Literature review
  - Formalization of annualization process

Literature Review of Annualization Techniques

Findings

- **~20 Aviation benefits documents**
  - Benefits Analysis Guidelines
  - Applications w/ annualization
- **Most annualizations at sub-national level**
  - Individual airports
  - Particular services

Examples

- **FAA Oceanic Systems Study**
- **Lincoln Lab Weather-Delay Reduction Study**

Annualization #1: FAA Oceanic Systems Study

- ***Purpose:*** **Benefits of new separation standards**
- ***Methodology:***
  - Simulation models of North Atlantic and Pacific Airspaces
  - Regression on (limited) simulation results, relating delay and traffic intensity for each route

$$\mathbf{f} = \ln(\textbf{delay per flight}) = \mathbf{a} + \mathbf{b}\ \ln(\textbf{Traffic/Capacity})$$

- ***Annualization:*** **Directly from regression function**

Annualization #2: MIT Lincoln Labs Aviation Weather Systems Study

- ***Purpose:*** **Quantifying Weather-Delay Reduction Benefits**
- ***Methodology*****: Analytical queuing model, usage observations (Benefits Blitz)**

MIT Lincoln Labs, cont'd

- ***Annualization*: For a given facility,**

Total Annual Benefit of Decision Z = Frequency of Convective Weather * Frequency of Decision *Z* per storm * Average Benefit of a Decision *Z*

Where:

- **Frequency of convective weather is established from historical weather data**
- **Frequency of decision is estimated from observation of usage**
- **Benefit per decision is calculated in queuing model for a random sample of occurrences of each type of benefit decision**

Formalization of the Annualization Problem

- Want to estimate the annual mean of some daily metric, *Y*, from a limited sample of days for which we have, or can obtain, the value for *Y*
- We know that *Y* is related to other variables, X, for which we have complete information. (We don't know what the relationship is, however.)

Issues

- **Choosing the days for which to measure *Y***
- **Using the measurement results to estimate the annual average**
  - Actual average: μ_Y
  - Estimate of average: $\hat{\mu}_Y$

Examples

Method	Choice of Measurement Days	Estimation Procedure
Simple random sampling	Choose *n* days at random.	Sample mean.
Cluster	Define *m* clusters of days that are similar with respect to *X* variables related to *Y*. Choose some days from each cluster.	Weighted average with weights based on proportion of annual days in each cluster.
Regression	Use *n* observations to estimate relationship *Y=g(X).*	Average *g(X)* over the whole year.

Criteria for Assessing Annualization Techniques

- **Unbiasedness:** $E(\hat{\mu}_Y) = \mu_Y$
- **Efficiency:** $VAR(\hat{\mu}_Y)$ **should be small**
- **It should be possible to establish rough confidence bounds on the true average**

Comparison of Cluster and Random Sampling Techniques

- **"Simulate" process of drawing a sample and making annual estimates using random sampling and cluster method**
- **Determine variance of estimates we would obtain if we did this multiple times**
- **The lower the variance, the more efficient (and better) the estimate**

Example

TRIAL	RANDOM SAMPLE	CLUSTER
1	11	19
2	20	20
3	29	21
variance	54	0.67
st-dev	7.35	0.82

Problem Set-up

- Days in year fall into *n* clusters.
- Researcher knows cluster assignments.
- Researcher wants to estimate annual average for *Y* from *n* daily measurements and can choose which days to measure.

Researcher Does not Know

- mean of *Y* for cluster *i*: μ_i
- variance of *Y* for cluster *i*: σ_i^2
- While the researcher does not know these parameters, we will use them to predict what the researcher will find

Simple Random Sampling

- **Researcher chooses *n* days at random, ignoring the information about the clusters**
- **Researcher uses sample mean to estimate annual mean**
- **What results will researcher obtain if she follows this procedure?**

Simple Random Sampling

- **The estimate obtained will be unbiased**
  - High and low estimates will cancel
  - $E(\hat{\mu}_Y) = \mu_Y$
- **The variance of the estimate will be:**

$$VAR(\hat{\mu}_Y) = \frac{\sum_i p_i(\mu_i{}^2 + \sigma_i^2) - (\sum_i p_i \mu_i)^2}{n}$$

Where p_i is proportion of days in cluster i

Cluster Approach

- **Researcher chooses *1* day at random from each cluster.**
- **Researcher uses weighted average (based of proportion of days in each cluster) estimate annual mean.**
- **What results will researcher obtain if she follows this procedure?**

Cluster Approach

- **The estimate obtained will be unbiased**
  - High and low estimates will cancel
  - $E(\tilde{\mu}_Y) = \mu_Y$
- **The variance of the estimate will be:**

$$VAR(\tilde{\mu}_Y) = \sum_i p_i{}^2 \sigma_i^2$$

Where p_i is proportion of days in cluster i

Random Sampling vs Cluster Approach

$$\frac{\sum_i p_i(\mu_i{}^2 + \sigma_i^2) - (\sum_i p_i \mu_i)^2}{n} \quad \text{vs} \quad \sum_i p_i{}^2 \sigma_i^2$$

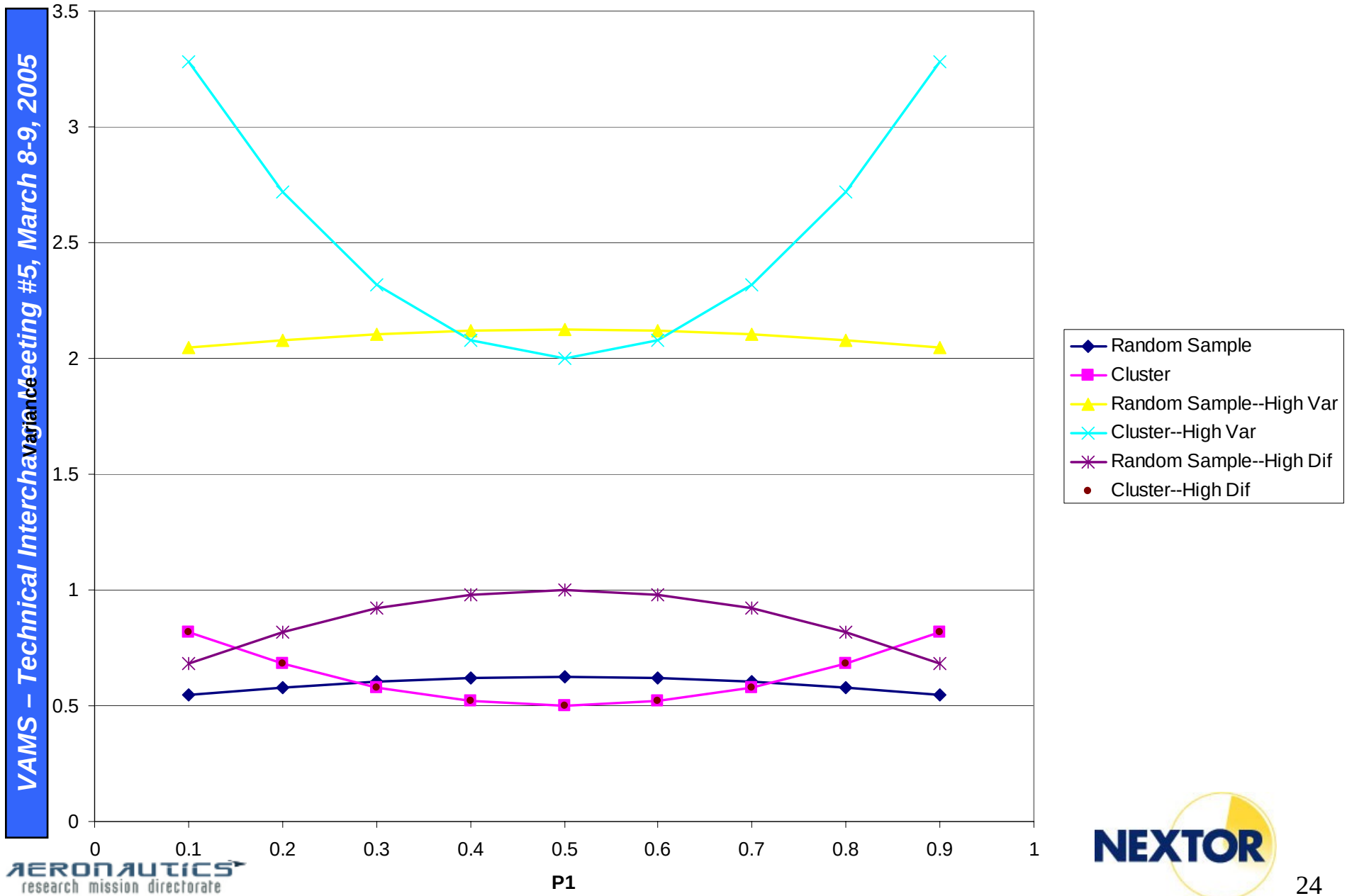
- **Cluster approach more efficient when**
  - p_i values are more equal
  - σ_i^2 are small
  - μ_i vary more among clusters

Numerical Example

- **Two clusters with equal variance:**

$$\sigma_1^2 = \sigma_2^2 = \sigma^2$$

- **Cluster 1 has mean 0 (i.e. $\mu_1=0$)**
- **Three cases**
  - $\mu_2=1,\sigma^2=1$
  - $\mu_2=1,\sigma^2=2$ “Hi Var”
  - $\mu_2=2,\sigma^2=1$ “Hi Dif”


Variance
3.5
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
P1
Random Sample
Cluster
Random Sample--High Var
Cluster--High Var
Random Sample--High Dif
Cluster--High Dif

Application to METRON Clusters

- **2002 days divided into seven clusters**
- **Compared random sample and cluster approaches for estimating annual averages for four performance metrics**
  - Average delay against schedule
  - On-time performance
  - Cancellation rate
  - Total NAS delay
- **Having complete daily data for these metrics we can determine which estimation method will be more efficient**
- **If a given method is consistently more efficient for these metrics, it is also likely be so for ACES generated ones**

Results

Metric	Annual Average	Rand. Sample St. dev	Cluster St. dev
Average delay (min)	9.76	1.50	1.16
On-time performance (%)	82.8	2.3	2.0
Cancellation rate (%)	1.30	0.41	0.36
Total delay (000 min)	204	32	24

- **Cluster method yields more efficient estimate for all four cases**
- **Differences are not huge**

Implications

- **Simulating one day per cluster is likely to produce slightly more efficient annualizations then simulating set of days drawn at random**
- **Cluster technique w/ one day per cluser does not allow uncertainty of estimates to be quantified**
- **Considering other possibilities**
  - Fewer clusters with multiple days simulated for (some) clusters
  - Clusters with more equal sizes
  - Regression-based methods